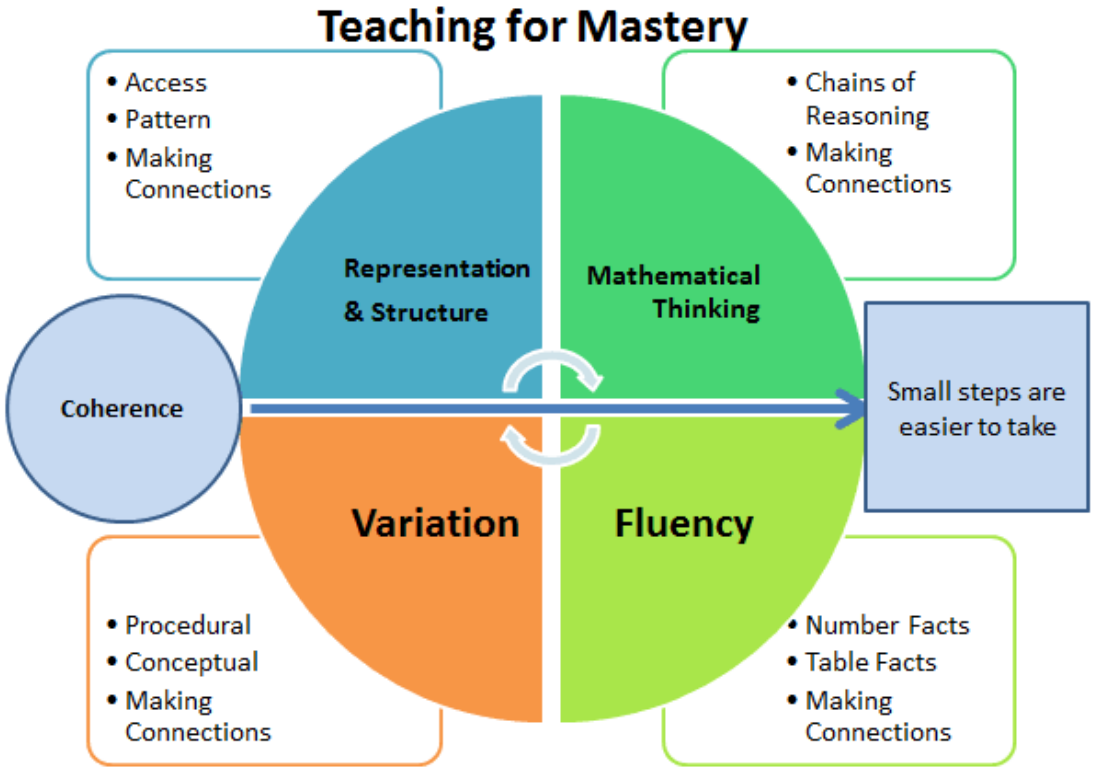
Rush Common
Maths Policy

Introduction

At Rush Common School we use a mastery approach to the teaching and learning of Maths.
The 2014 National Curriculum states:
The expectation is that most pupils will move through the programmes of study at broadly the same pace.
Pupils who grasp concepts rapidly should be challenged through being offered rich and sophisticated problems before any acceleration through new content.
Those who are not sufficiently fluent with earlier material should consolidate their understanding, including through additional practice, before moving on.

Five Big Ideas[1]



1. NCETM

Our teaching for mastery is underpinned by the **NCETM's 5 Big Ideas**. Opportunities for **Mathematical Thinking** allow children to make chains of reasoning connected with the other areas of their mathematics. A focus on **Representation and Structure** ensures concepts are explored using concrete, pictorial and abstract representations; children actively look for patterns as well as specialising and generalising whilst problem solving. Coherence is achieved through the planning of small connected steps to link every question and lesson within a topic. Teachers use both procedural and conceptual **Variation** within their lessons and there remains an emphasis on **Fluency** with a focus on number and times table facts.

Teaching Principles

1. Teachers believe in the importance of Maths and that the vast majority of children can succeed in learning Maths in line with national expectations.

2. The whole class is taught Maths together, with no differentiation by acceleration to new content. We do not group children by ability. The learning needs of individuals are addressed through careful scaffolding, questioning and appropriate rapid intervention where necessary, to provide the appropriate support and challenge.

3. The reasoning behind mathematical processes is emphasized. Teacher/pupil interaction explores how answers were obtained as well as why the method worked and what might be the most efficient strategy.

4. Precise mathematical language is used by teachers so that mathematical ideas are conveyed with clarity and precision. We value 'Maths talk' and children get lots of opportunity to talk about and evaluate their Maths during lessons. Children are encouraged to speak in full sentences and are given stem sentences to assist them with this.

5. Conceptual variation and procedural variation are used in teaching. This helps to present the Maths in ways that promote deep, sustainable learning. Conceptual variation is where the concept is varied and examples are shown of what that concept is and isn't (for example a triangle or a quarter.) Procedural variation is where different procedures and/or representations are used to bring about understanding. For example, teachers may collect several solutions for a problem (some right, some wrong) before guiding the class towards the most efficient method. It also involves highlighting the essential features of a concept or idea through varying the non-essential features. Procedural variation provides the opportunity to focus on relationships, not just the procedure to make connections between problems using one problem to work out the next.

6. Sufficient time is spent on key concepts to ensure learning is well developed and deeply embedded before moving on.

Lesson Design

1. Lessons are sharply focused with one new objective introduced at a time.

2. Potential misconceptions are identified in advance and strategies to address them planned. Key questions are planned, to challenge thinking and develop learning for all pupils. Mistakes are valued and used to move learning forward.

3. Independent practice includes starting with a core task. Tasks include reasoning, problem solving and higher-order thinking activities. Children who require more practice will be given consolidation activities at an appropriate challenge level. Pupils who grasp concepts rapidly are challenged through being offered rich and sophisticated problems.

4. The use of high quality materials and tasks (White Rose, Maths No Problem NRICH, NCETM Mastery Assessment materials) to support learning and provide access to the mathematics is integrated into lessons.

5. There is regular interchange between concrete/contextual ideas and their abstract/symbolic representation.

6. Making comparisons is an important form of developing deep knowledge. The questions “What’s the same, what’s different?” are often used to draw attention to essential features of concepts. E.g. What is a triangle? What it isn't... What it is..

7. Formative assessment is carried out throughout the lesson; the teacher regularly checks pupils’ knowledge and understanding and adjusts the lesson accordingly. This forms part of the mastery learning instructional process.

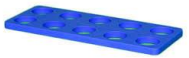
Calculation

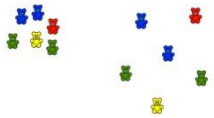
Introduction:

Children are introduced to the processes of calculation through practical, oral and mental activities. As they begin to understand the underlying ideas, they develop ways of recording to support their thinking and calculation methods, so that they develop both **conceptual understanding** and **fluency** in the fundamentals of mathematics. Whilst interpreting signs and symbols involved with calculation, orally in the first instance, children use both manipulatives as well as pictorial representations (potentially as part of a **Concrete-Pictorial-Abstract – CPA – approach**) to support their mental and written methods of calculation. As children's mental methods are strengthened and refined, they begin to work more efficiently, which will support them with using succinct written calculation strategies as they are developed.

<u>From Early Years to Year 1:</u>

There are fundamental concepts that it is important for children to develop an early understanding of as building blocks to future learning in maths, including that linked to calculation. A selection of the skills include:

- Ordinality – 'the ordering of numbers in relation to one another' – e.g. (1, 2, 3, 4, 5…)
- Cardinality – 'understanding the value of different numbers' – e.g. (7 =  17 =  +  14 = 
- Equality – 'seven is the same total as four add three' – e.g.  = 
- Subitising – 'instantly recognising the number of objects in a small group, without counting them' – e.g.  → five
- One-to-one correspondence – e.g. 



- Conservation of number – ‘recognising that a value of objects are the same, even if they are laid out differently’ – e.g.

- Concept of zero  3 + 0 = 3

- Counting on and back from any number – e.g. ‘five add three more totals eight’ ‘ten take away three totals seven’

The ability to calculate mentally forms the basis of all methods of calculation and has to be maintained. In the 2018 national Key Stage 1 SATs tests, every one of the named mental maths strategies below was assessed, whilst many also featured in a less explicit manner in the Key Stage 2 SATs tests, hence highlighting the need for each method to be taught explicitly. A good knowledge and ‘feel’ for numbers, is the product of structured practice through progression in relevant practical maths experiences alongside visual representations.

By the end of Year 6, children should be equipped with efficient mental and written calculation methods, which they use fluently. Decisions about when to progress should always be based on the security of pupils’ understanding and their readiness to move ahead to the next stage. At whatever stage in their learning, and with whatever written method is being used, children’s strategies must still be underpinned by a secure understanding and knowledge of number facts that can be recalled fluently with flexibility.

The overall aims are that when children leave primary school they:

- Are able to recall number facts with fluency, having developed conceptual understanding through being able to visualise key ideas – such as those related to place value, through experience with practical equipment and visual representations;
- Make use of diagrams (including the bar model) and jottings to help record / reason through stages of thinking when using mental methods that generate more information than can be kept in their heads;
- Have an efficient, reliable, written method of calculation for each number operation that they can apply with confidence when undertaking calculations that they cannot carry out mentally;
- Are able to make connections between all four number operations, understanding how they relate to one another, as well as how the rules and laws of arithmetic can be applied.

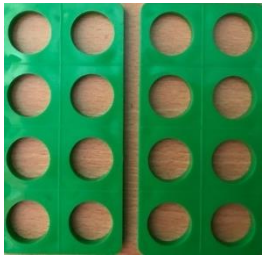
.

Addition:

Mental calculation strategies for addition and subtraction:

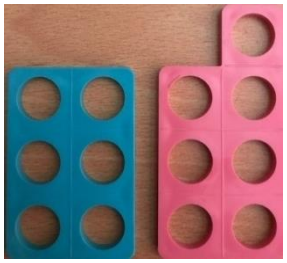
All mental calculation strategies need to be taught explicitly using a Concrete – Pictorial – Abstract (CPA) approach in every year group, for example, using decimals in Key Stage 2. The following ideas can be adjusted so that they are accessible to all children. The NCETM, 2015, state that, 'a pupil really understands a mathematical concept, idea or technique if he or she can represent it in a variety of ways.'

Doubles: 8 + 8 = 16



8 + 8 is connected to 8 X 2

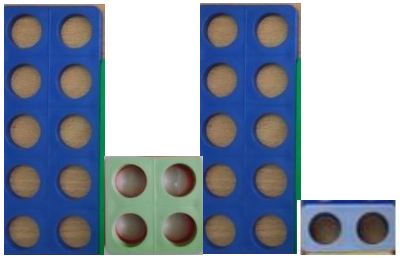
Near doubles: 6 + 7 = 13



6 + 7 is commutative with 7 + 6

Number bonds: 7 + 3 = 10

Partitioning: 14 + 12 = 26



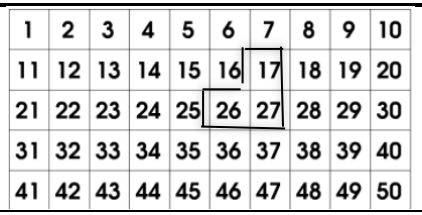
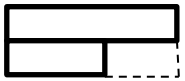
Bridging: 7 + 5 = 12

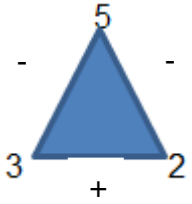
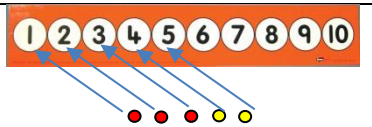
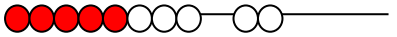
To begin: 7 + 3 = 10
Then: 10 + 2 = 12

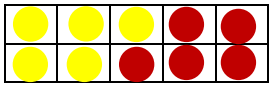
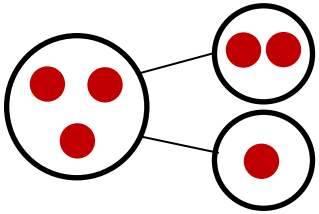
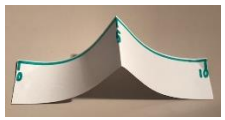
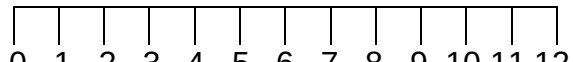
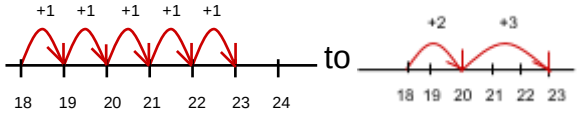
Adjusting: 16 + 9 = 25

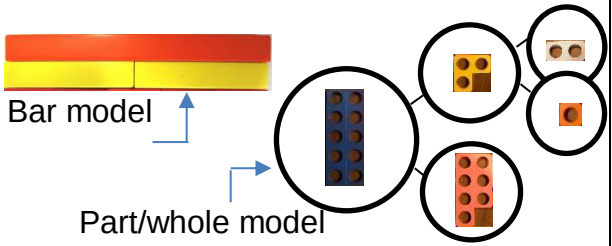
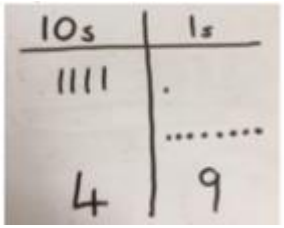
To begin: 16 + 10 = 26

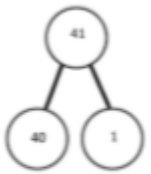
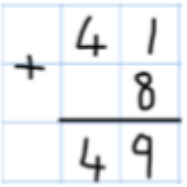
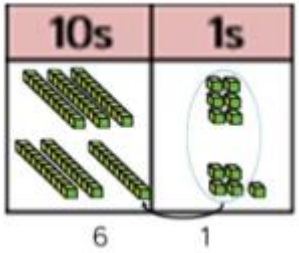
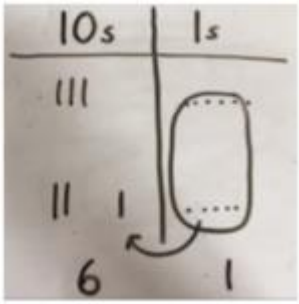
Then: 26 – 1 = 25

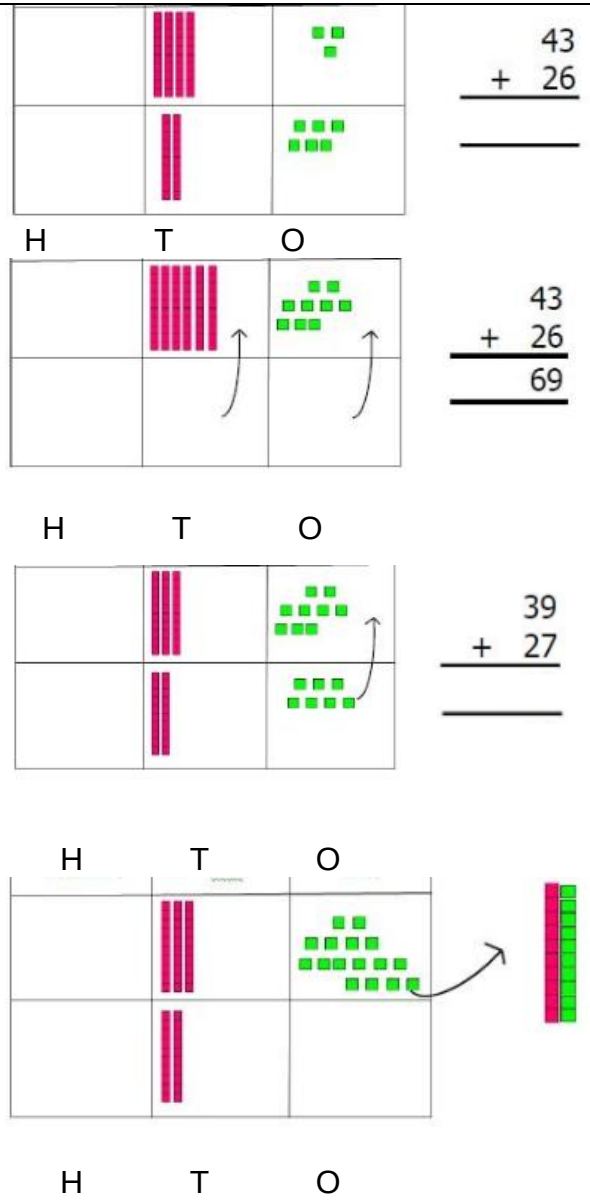
				
	Finding the difference: 10 – 6 = 4   David has 10 sweets, whilst Chloe has six sweets. How many more does David have than Chloe? 	**Reordering:** 8 + 7 + 2 = 17 e.g. calculating numbers in a different order To begin: 8 + 2 = 10 Then: 10 + 7 = 17 	

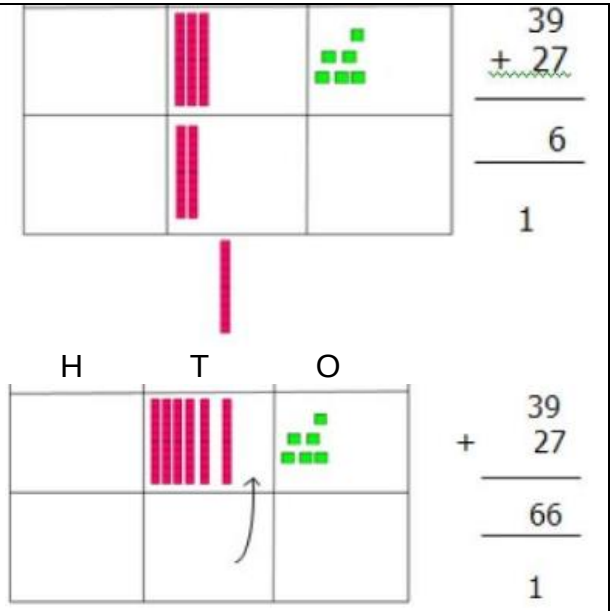
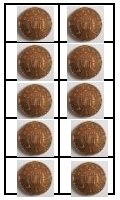
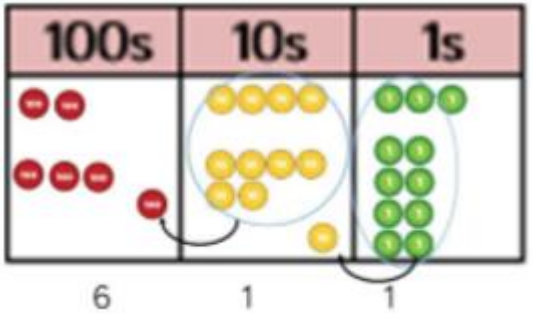
	Counting	**Mental maths strategies & linked concepts**	**Rapid recall**	**Written calculation & appropriate models/images to support conceptual understanding and varied fluency**	
Stage 1:	Count in ones to and across 100 forwards and backwards starting from 0, 1 and other numbers. Count in multiples of two, five and ten using a counting stick	<u>Explicitly teach every mental maths strategy detailed above.</u> Pupils use apparatus to explore addition as the inverse of subtraction. 	Rapid recall of all pairs of numbers totalling numbers up to 20. Use structured apparatus – i.e. Numicon, tens frames, abaci, etc.	**Combining two groups:** • Teachers model how to line up counters/objects on a number track before counting on. This is a precursor to use of a fully numbered number-line. • Children develop a mental picture of the number system for use with calculation. A range of key models and images support this, alongside practical equipment.	 3 + 2 = 5 'Three plus two is the same as five'  'Eight add two more makes ten'

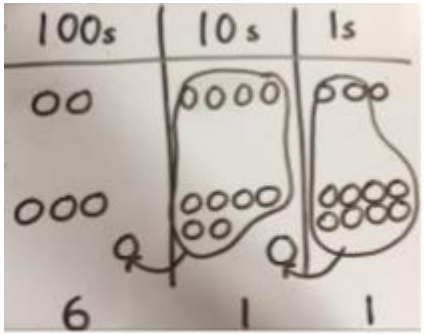
	set up as a number track.	‘Four add one is the same as five’ 		**Whole / part-whole model:** • The concept of a whole / part-whole model is introduced.	 ‘Four add one more is the same as five’  Tens frame  Bar model  Part/whole model
Stage 2:	Continue practising above skills. Count in steps of 2, 3 and 5 forwards and backwards to and from zero using a counting stick set up as a number line. Count in tens from any number – link to coins in a	<u>Explicitly teach every mental maths strategy detailed above.</u> Round numbers to the nearest 10, for example, by illustrating on a number line that is drawn on a folded strip of paper. 	Recall addition facts for all numbers to 20.	**Counting on from the largest number:** • Children begin to use number lines to support their own calculations, initially counting on from the largest number in ones before beginning to work more efficiently. **Reordering calculations to apply use of mental maths strategies:** • Children reorder ‘strings’ of numbers to apply their understanding of mental maths strategies, including doubles and number bonds,	Number line with all numbers labelled  18 + 5   Questions such as: ‘How might I rearrange these to find the total?’ are asked.

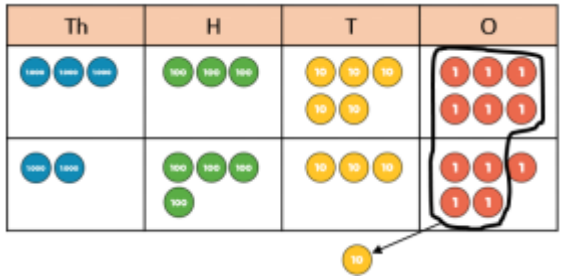
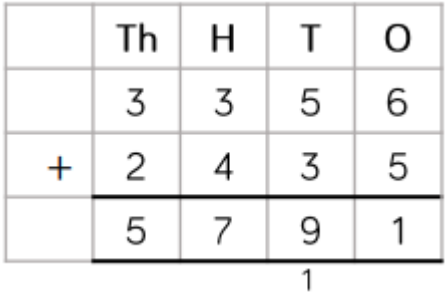
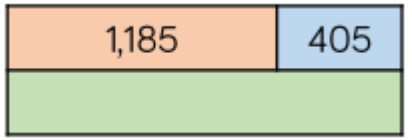
	piggy bank as well as a number square.			e.g. 6 + 7 + 4 reordered to 6 + 4 = 10 and then 10 + 7 = 17. Jottings are used to help keep track of thinking. **Whole / part-whole model:** • The concept of a whole / part-whole model is reinforced and extended. **Tens and ones + ones** • Continue to develop understanding of partitioning and place value. Represent the numbers using Dienes. • Children to draw the base 10 e.g by drawing rectangles for tens and squares for ones or lines for 10s and dot/crosses for ones. • Add the ones and then the tens.	 41 + 8  

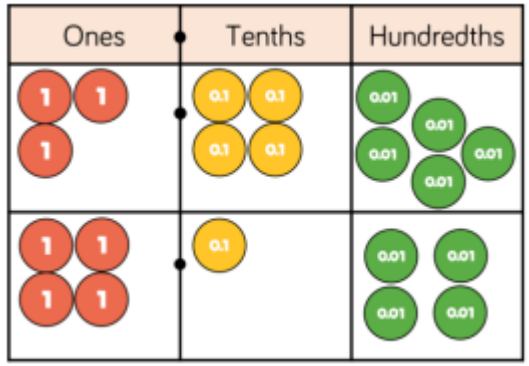
				Tens and ones + tens and ones • Continue to develop understanding of partitioning and place value. Represent the numbers using Dienes. • Children to represent the Dienes in a place value chart. **Column Addition** <u>Without regrouping</u> <u>Step 1</u> • Dienes are used to represent both numbers. <u>Step 2</u> • Starting with the ones, the Dienes are combined to form a total. The resulting totals are	41 + 8  1 + 8 = 9 40 + 9 = 49  36 + 25   H T O

				written simultaneously in the column layout. <u>With regrouping</u> <u>Step 1</u> • Dienes are again used to represent both numbers. <u>Step 2</u> • Starting with the ones, the Dienes representations are combined to form a total. Ten of the ones must be exchanged for a tens stick. <u>Step 3</u> • This leaves six ones in the ones column to be recorded using the written method. The exchanged tens stick is now placed underneath the tens column. This is mirrored by recording the one ten under the line in the written calculation. <u>Step 4</u>	

				Finally, the tens are combined to give a resulting total of tens which is recorded in the written calculation.	
Stage 3:	Continue practising above skills. Count forward and backwards from 0 in multiples of 4, 8, 50 and 100. Count on 10 or 100 from any two-digit number. Count up and down in	Reinforce partitioning and bridging through multiples of 10, plus adjusting when adding 11 or 9. Use structured apparatus to understand that subtraction undoes addition and link with	Connect pairs totalling ten to pairs of multiples of 10 totalling 100.  Use 10ps in tens frame.	**Column addition** Continue to make links with earlier models and images, including the number line. **Use of place value counters to add HTO + TO and HTO + HTO** When there are 10 ones in the 1s column, we exchange for 1 ten. When there are 10 tens in the 10s column we exchange for 1 hundred.	

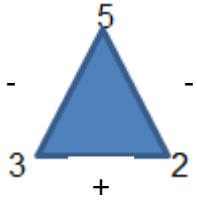
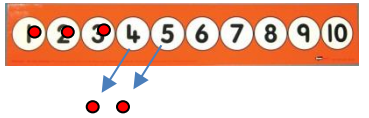
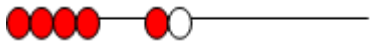
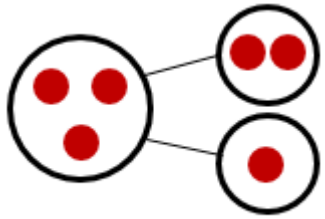
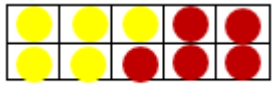
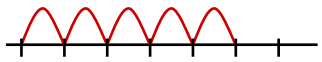
	tenths. Link to a counting stick as before, whilst deriving number facts.	inverse number operations.	Recall pairs of two-digit numbers with a total of 100, i.e. 32 + ? = 100.	Children to represent the counters in a place value chart, circling when they make an exchange. The addition is then shown using the formal written method.	 243 +368 611 1 1

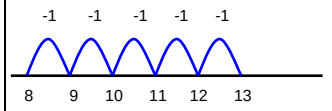
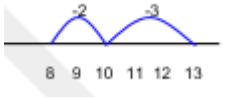
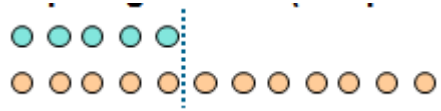
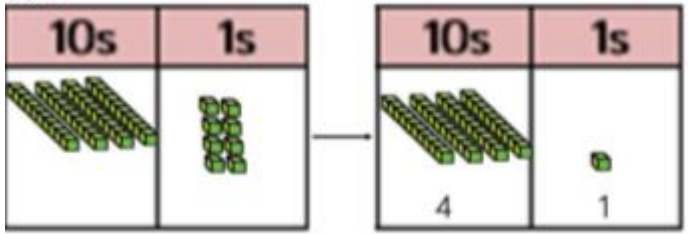
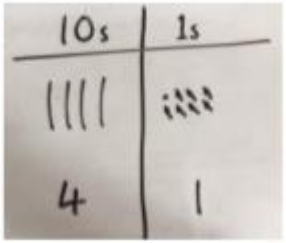
Stage 4:	Continue practising previous skills. Count forwards and backwards from 0 in multiples of 6, 7, 9, 25 and 1000 using counting sticks, number lines, number squares, etc. Count up and down in tenths, hundredths and simple fractions using models and images, plus Dienes / pixie Dienes equipment and a counting stick.	Bridging through 60 for time, i.e. 70 minutes = 1 hour and 10 minutes. Rounding any number to the nearest 10, 100 or 1000. Rounding numbers with one decimal place to nearest whole number. Explore inverse as a way to derive new facts and to check accuracy of answers.	As above. Use known facts and place value to derive new ones, i.e. 'If I know 8 + 3 = 11, I also know 0.8 + 0.3 = 1.1 and 8/100 + 3/100 = 11/100.' Sums and differences of pairs of multiples of 10, 100 or 1000. Addition doubles of numbers to 100. Pairs of fractions totalling one.	**Column addition** Continue to make links with earlier models and images, including the number line. **Use of place value counters to add ThHTO + HTO and ThHTO + ThHTO** Continue using bar models to represent calculations	3356 + 2435   

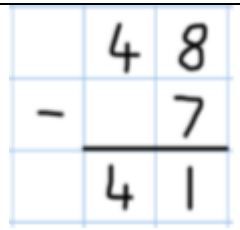
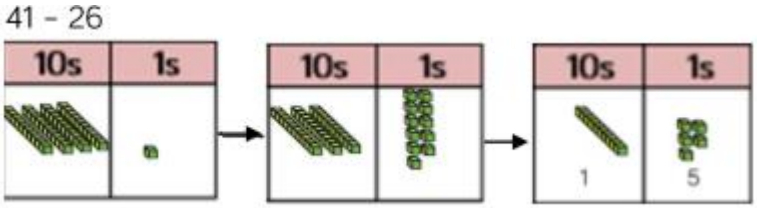
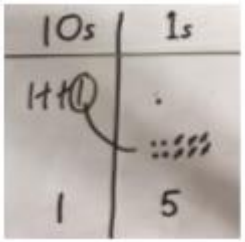
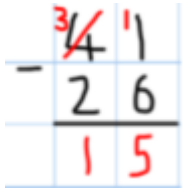
Stage 5:	Count forwards and backwards in steps of powers of 10 for any given number up to one million. Continue to count forwards and backwards in simple fractions. Count forward and backwards in appropriate decimals and percentages.	Use apparatus and knowledge of place value to add decimals, i.e. 3.4 + 2.5 = 5 + 0.9  Reorder increasingly complex calculations, i.e. 1.7 + 2.8 + 0.3 = 1.7 + 0.3 + 2.8 Compensating – i.e. 405 + 399 → add 400 and then subtract one.	Continue to practise previous stage and make links between known facts and addition pairs for fractions, percentages and decimals Doubles and halves of decimals, i.e. half of 5.6, double 3.4. Sums and differences of decimals, i.e. 6.5 + 2.7	Continue to use previous strategies to add numbers with more than 4 digits Use place value counters to add decimals	3.45 + 4.14  3 . 4 5 + 4 . 1 4
Stage 6:	Continue to practise previous skills. Count forwards and backwards in simple fractions, decimals and percentages.	Bridging through decimals, i.e. 0.8 + 0.35 = 0.8 + 0.2 + 0.15 using empty number lines. Partitioning using near doubles, i.e. 2.5 + 2.6 = 5 + 0.1	Using children's confident recalling of basic facts to 20/100 and deriving facts using place value, make links between decimals,	Continue to use previous strategies to add numbers with more than 4 digits. Use previous strategies to add decimals.	

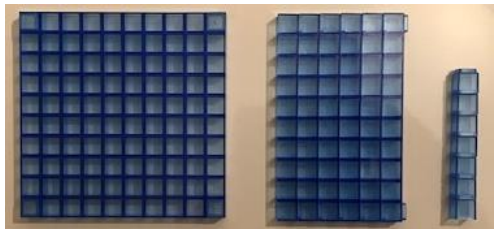
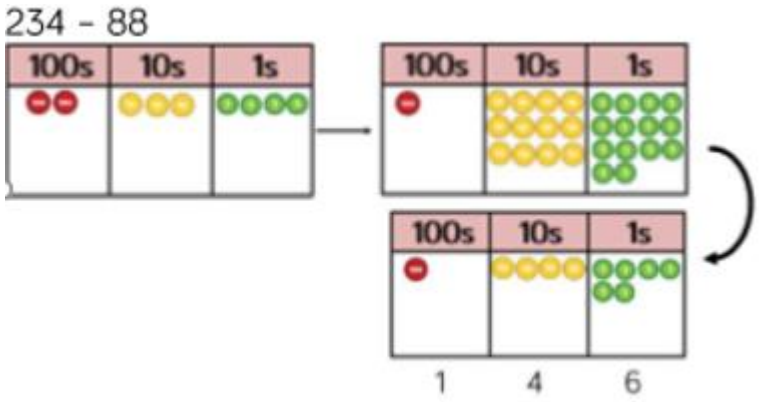
		Reorder decimals, i.e. 4.7 + 5.6 – 0.7 …as… 4.7 – 0.7 + 5.6 = 4 + 5.6.	fractions and percentages. i.e. 1 + 19 10 + 190 100 + 1900 Question: What do you notice?		

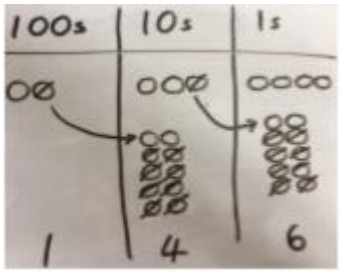
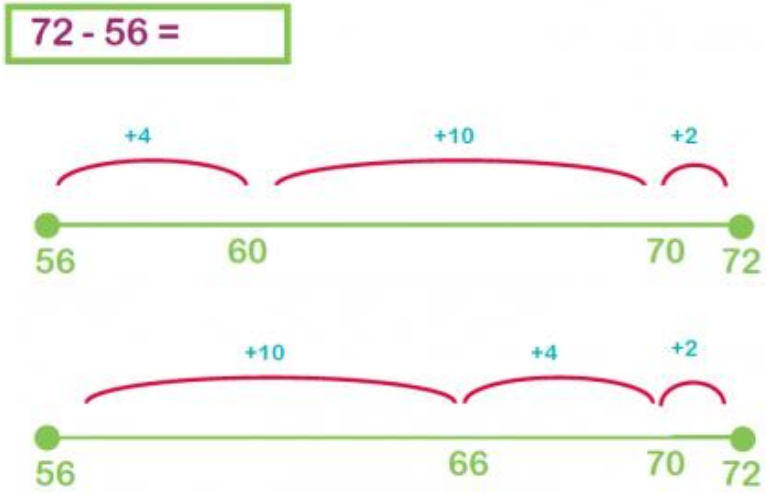
Subtraction:

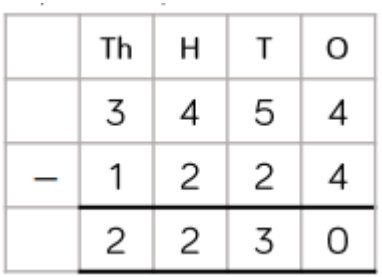
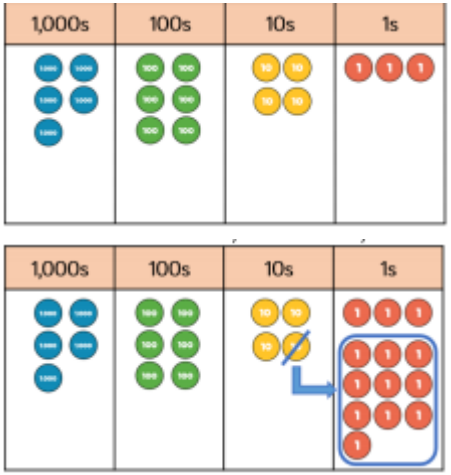
	Counting	Mental strategies	Rapid Recall	Written calculation and appropriate models and images to support conceptual understanding	
Stage 1:	Count in ones to and across 100, forwards and backwards starting from 0, 1 and other numbers. Count in multiples of two, five and ten.	<u>Explicitly teach every mental maths strategy detailed above.</u> Pupils use apparatus to explore addition as the inverse of subtraction:  'One less than five is four' 	Rapid recall of subtraction facts for numbers up to 10. Use structured apparatus, i.e. Numicon, tens frames, abaci etc. 	**Subtraction as taking away from a group:** • Teachers model how to remove counters/objects and count back on a number track. This is a precursor to use of a fully numbered number-line. **Whole / part-whole model:** • The concept of a whole / part-whole model is introduced. • Children begin to use number lines to support their own calculations, initially counting back in ones before beginning to work more efficiently.	 'Five minus two totals three'  'Six take away two leaves four'   Cherry model 'One less than six is five'  Tens frame  Bar model Number line with all numbers labelled 14-6 =8  

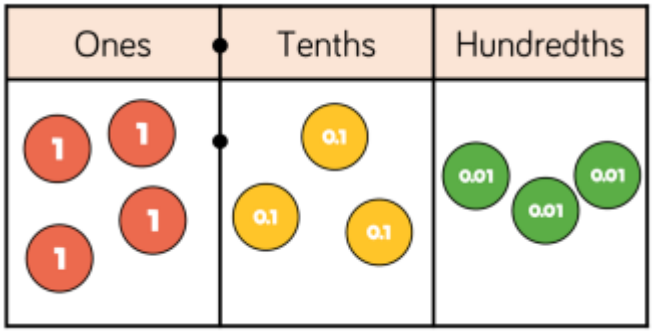
Stage 2:	Continue practising above skills. Count in steps of 2, 3 and 5, forwards and backwards to and from zero. Count in tens from any number – link to coins in a piggy bank as well as a number square.	<u>Explicitly teach every mental maths strategy detailed above.</u>	Recall subtraction (and addition) facts for all numbers to 20.	**Taking away:** Children continue to use numbered lines and also draw their own number lines **Finding the difference:** • Teachers model how to find the difference when two numbers are relatively 'close together.' **Column method** using Dienes (no exchange) Children to represent Base 10 pictorially.	13 – 5 = 8  to 13 – 5 = 8  Comparing two sets to find the difference.  48-7  

				Progressing to column method or children could count back.	
				Column method using Dienes (with exchange)	
				Represent the Dines pictorially, remembering how to show the exchange.	
				Formal column method. Children must understand that when they have exchanged the 10 they still have 41 because 41 = 30 + 11	

Stage 3:	Continue practising above skills. Count from 0 in multiples of 4, 8, 50 and 100. Count on and back by 10 or 100 from any two digit number. Link to counting stick counting forwards and backwards flexibly. Count up and down in tenths – linking to visual image.	Reinforce partitioning and bridging through multiples of 10, plus adjusting when subtracting 11 or 9. Use structured apparatus to understand that subtraction undoes addition and link with inverse number operations.	Connect subtractions from ten to subtractions from multiples of 10 totalling 100.  Use 10ps in tens frame. Subtract two digit numbers from 100 i.e. ? = 100 - 78	**Taking away:** • When teaching children about reduction, highlight the importance of only partitioning one number. Column method using place value counters	Subtraction by partitioning with use of manipulatives 167 – 24 = 143 20 4  In either order… To begin: 167 – 20 = 147  Then: 147 – 4 = 143  

				Represent the place value counters pictorially, remembering how to show the exchange. Formal column method. Children must understand what has happened when they have crossed out the digits. **Finding the difference:** Children should note that finding the difference is often the most efficient way of solving a subtraction problem when two numbers are close together.	 Finding the difference on a number line: $\begin{array}{r} {}^{2}2\,{}^{1}3\,4 \\ -\ 8\,8 \\ \hline 6 \end{array}$ 

Stage 4:	Continue practising of previous skills. Count forwards and backwards from 0 in multiples of 6, 7, 9, 25 and 1000 using counting sticks, number lines, number squares, etc. Count up and down in tenths, hundredths and simple fractions using models and images, i.e. Dienes / Pixie Dienes equipment, counting stick, ITPs.	Bridging through 60 for time, i.e. 70 minutes = 1 hour and 10 minutes Rounding any number to the nearest 10, 100 or 1000. Rounding numbers with one decimal place to nearest whole number. Explore inverse as a way to derive new facts and to check accuracy of answers.	As above. Use known facts and place value to derive new ones, i.e. 'If I know 11 - 3 = 8, I also know 1.1 - 0.3 = 0.8 and 8/100 - 3/100 = 5/100.' Sums and differences of pairs of multiples of 10, 100 or 1000. Subtraction of fractions totalling 1, i.e. 1 – 0.3 = 0.7	Column method using place value counters (no exchange) Column method using place value counters (with exchange)	  

				Finding the difference: Finding the difference continues to be highlighted where the two numbers are close together – using a number line on a strip of paper.	 
Stage 5:	Count forwards and backwards in steps of powers of 10 for any given number up to one million. Continue to count forwards and backwards	Use apparatus and knowledge of place value to subtract decimals, i.e. 3.8 - 2.5 = 1.3 Reorder increasingly complex calculations, i.e. 1.7 – 0.5 – 0.7 = 1.7 – 0.7 – 0.5. Compensating – i.e. 405 - 399 →	Continue to practise previous stage and make links between known facts and addition pairs for fractions, percentages and decimals.	Continue to use previous strategies to subtract numbers with more than 4 digits • Use place value counters to subtract decimals	Use the place value chart to find the answer to 4.33 – 2.14 

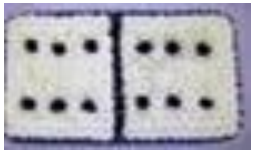
	in simple fractions. Count forward and backwards in appropriate decimals and percentages.	subtract 400 and then add 1.	Doubles and halves of decimals, i.e. half of 5.6, double 3.4. Sums and differences of decimals, i.e. 6.5 + 2.7		$\begin{array}{r} 4.33 \\ -\ 2.14 \\ \hline \\ \hline \end{array}$
Stage 6:	Continue to practise previous skills. Count forwards and backwards in simple fractions, decimals and percentages.	Bridging through decimals, i.e. 1.5 – 0.8 = 1.5 – 0.5 then – 0.3 using empty number line.	Using children's confident recalling of basic facts to 20/100 and using place value, make links between decimals, fractions and percentages. 19 – 1 = 190 – 10 = 1900 – 100 = 1.9 – 0.1 = Question: What do you notice?	Continue to use previous strategies to subtract numbers with more than 4 digits. Use previous strategies to subtract decimals.	

Multiplication:

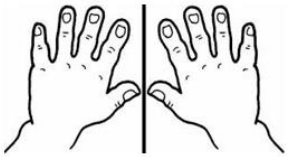
Mental calculation strategies for multiplication and division:

Doubling and halving:

Double six is 12…



Double five is ten…



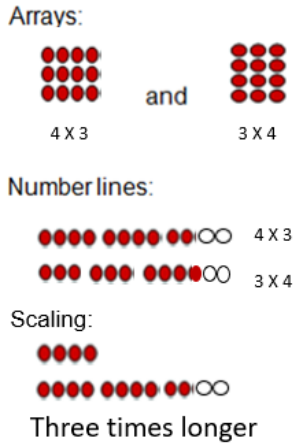
Double 16 can be calculated by working out…

Double ten → 20

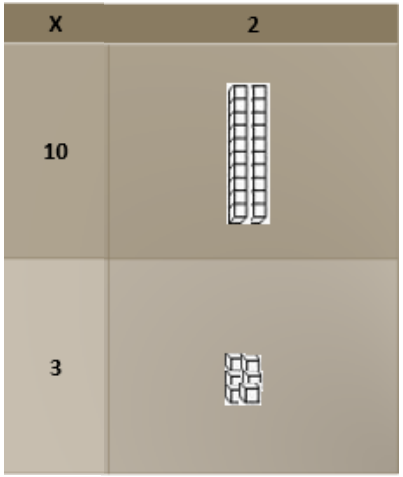
Double six → 12

With links to finding four-times a given value and finding a quarter of a value.

Knowing multiplication and division facts to 12 X 12:



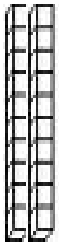
Multiplying a teen number by one-digit number:



Multiplying and dividing by multiples of ten:

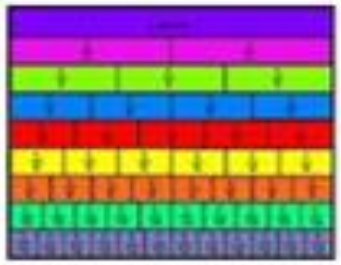
20 X 10 = 200

Hundreds	Tens	Ones
	2	0

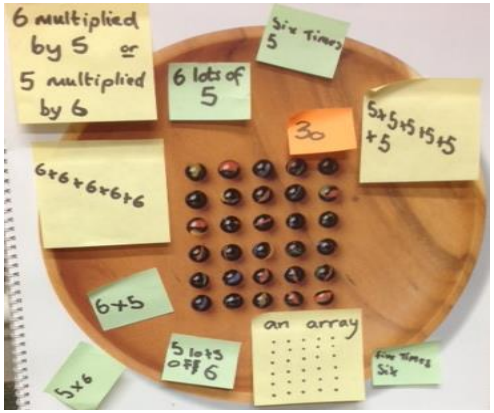


‘Add a place value holder’

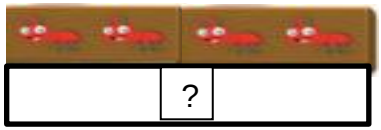
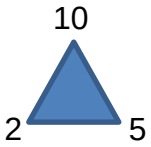
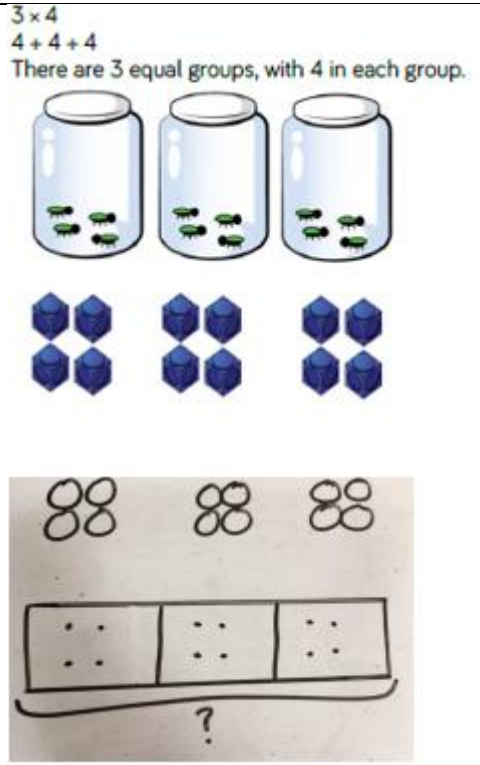
Identifying fractions, decimals and percentages:

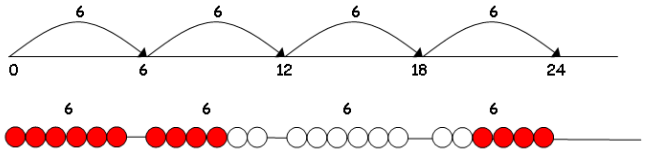


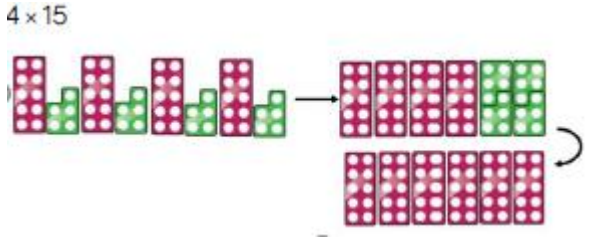
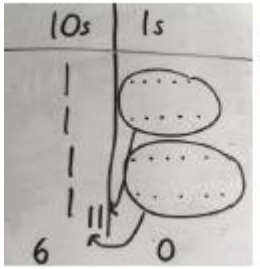
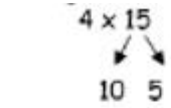
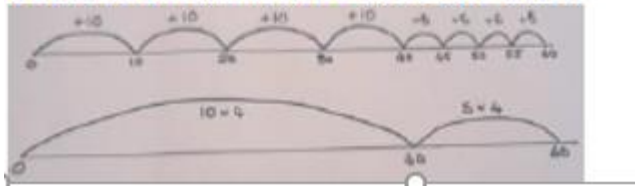
Milk the maths…

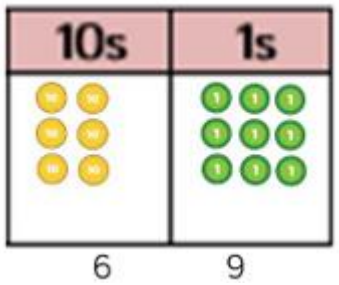
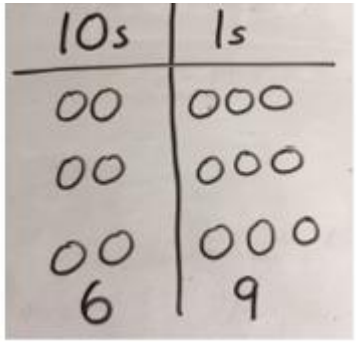
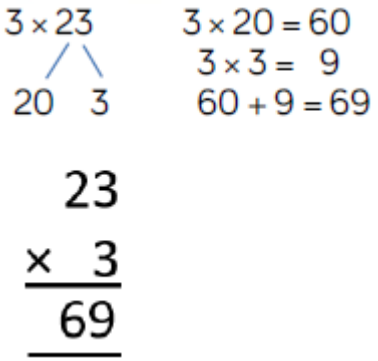


...by allowing children to make connections between number facts.

	Counting	Mental strategies	Rapid recall	Written calculation and appropriate models and images to support conceptual understanding	
Stage 1:	Count forwards and backwards in 2s, 5s and 10s	Doubling up to six and then ten whilst using related models and images.	Derive/recall doubles up to five and derive/recall halves up to ten.	**Developing early conceptual understanding of multiplication (grouping):**	Use objects, pictorial representations and arrays to show the concept of multiplication:  Early bar model 
Stage 2:	Count forwards and backwards in 2s, 3s, 5s and 10s from zero.	Begin to understand and use inverse number operations:  Stories are used alongside a triad to help children understand links between number operations, e.g. "There are five pencils in two packs, which means that there are ten pencils altogether." Doubling is reinforced using a whole/part-whole model: 	Derive/recall doubles up to ten and derive/recall halves up to twenty. Recall & use multiplication facts for the 2X, 5X and 10X-tables. Learn what happens when a number is multiplied by zero or one.	**Understanding multiplication as repeated addition:** • Investigate multiplication as repeated addition, so that the law of cummutativity is understood. • Children to represent the practical resources in a picture and use a bar model..	

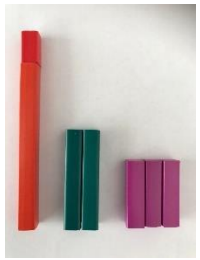
				Arrays: • Represent with apparatus and pictorially. • Connect related facts with both array and repeated addition images. **Repeated addition on the number line** linked with manipulatives:	$3 \times 4 = 12$ $4 + 4 + 4 = 12$ 5 X 3  and 3 X 5  6 X 4 = 24  So: ‘Six multiplied by four’ …or… ‘Six taken four times.’

Stage 3:	Counting forwards and backwards in 2s, 3s, 4s, 5s, 8s and 10s from zero. Count up and down in tenths.	Use doubling to make connections between the 2X, 4X and 8X-tables. Understand that multiplication can be undertaken by partitioning numbers, e.g. 12 X 4 = 10 X 4 + 2 X 4 Introduce the structure of scaling: e.g. Find a ribbon hat is 4 times as long as the blue ribbon  2cm 8cm	Recall and use multiplication facts for the 2X, 3X, 4X, 5X, 8X and 10X tables.	Partition to multiply using Numicon and Dienes. Children to represent the concrete manipulatives pictorially. Children to show the steps they have taken.	   10 x 4 = 40 5 x 4 = 20 40 + 20 = 60 A number line can also be used 

				Formal column method with place value counters (Dienes can also be used.) Children to represent the counters pictorially. Children to record what they are doing to show understanding.	3 X 23   

<table>
<tr>
<td>Stage 4:</td>
<td>Counting forwards and backwards in 2s, 3s, 4s, 5s, 7s, 8s, 10s, 25s and 1000s from zero.

Count up and down in tenths and hundredths.</td>
<td>Derive factor pairs of numbers using models and images, e.g. Cuisenaire
1 and 12
2 and 6
3 and 4

Use reordering to multiply three numbers.

Children learn about the associative law:
(9 X 5) X 10 = (10 X 5) X 9</td>
<td>Recall & use multiplication facts for all times-tables up to 12 X 12.</td>
<td>Relate multiplying a 3 or 2-digit by 1-digit number with arrays to support using a written method:

Use place value counters to support written multiplication.</td>
<td>Relate multiplying a 3/2-digit by 1-digit number, whilst refining the written notation used.

114 X 2 = 228

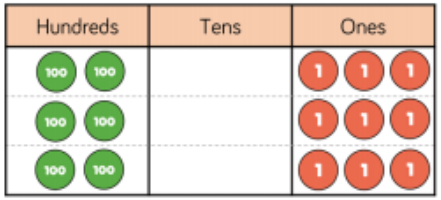
<table>
<tr><td>X</td><td>100</td><td>10</td><td>4</td></tr>
<tr><td>2</td><td>  </td><td>  </td><td>  </td></tr>
</table>

114 X 2 =

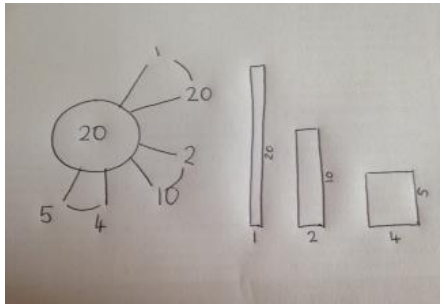
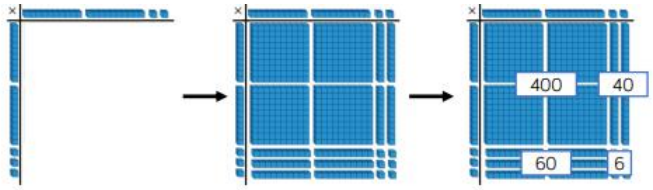
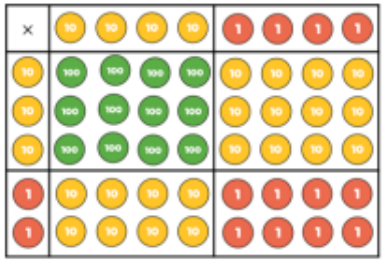
100 X 2 = 200
10 X 2 = 20
4 X 2 = 8
= 228

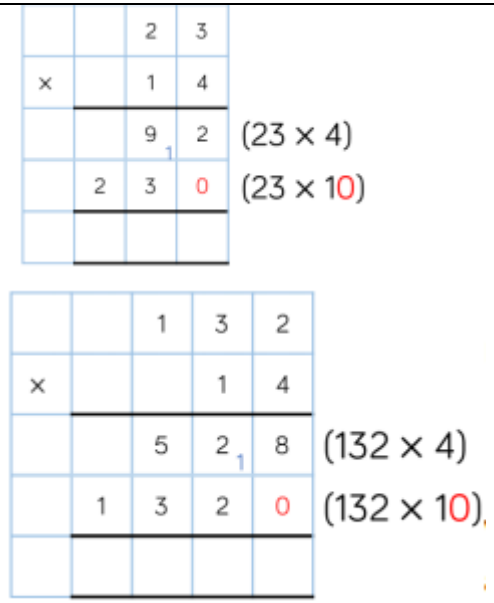
114
x 2
228

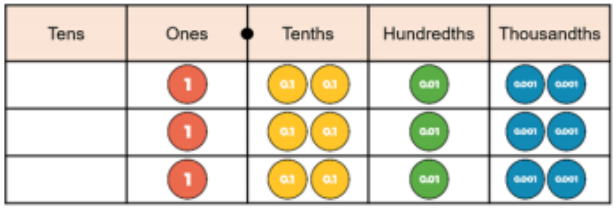
203 X 3

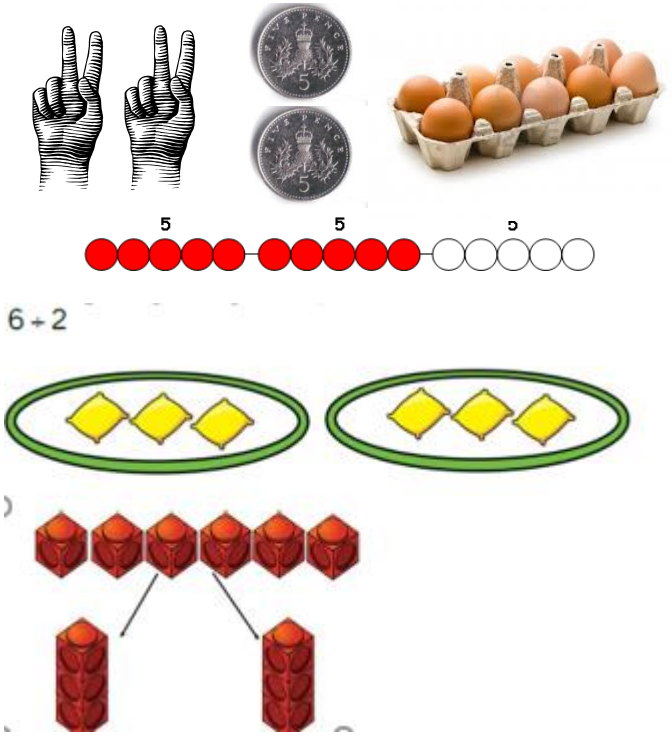
 
</td>
</tr>
</table>

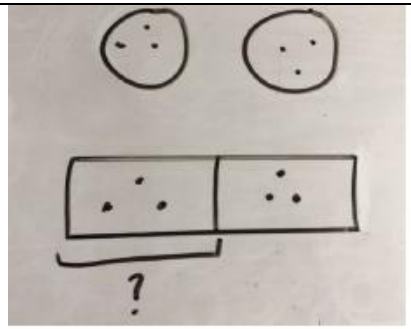
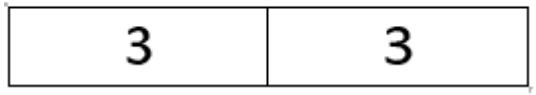
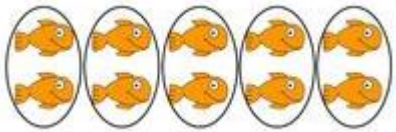
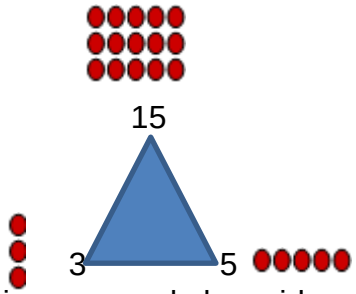
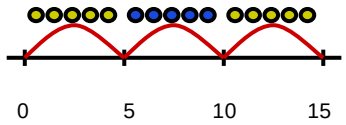
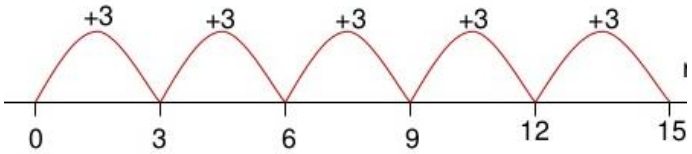
Stage 5:	Counting forwards and backwards in 2s, 3s, 4s, 5s, 6s, 7s, 8s, 9s, 10s, 25s and 1000s from zero.	Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers. 	Recall & use multiplication facts for all times-tables up to 12 X 12.	Children use Dienes to represent the area model of multiplication, which will enable them to see the size and scale linked to multiplying. Children will then move on to representing multiplication more abstractly with place value counters. They then progress to the formal written method: Discuss the importance of zero as a place holder.	23 X 22  44 X 32  × 40 4 30 1,200 120 2 80 8

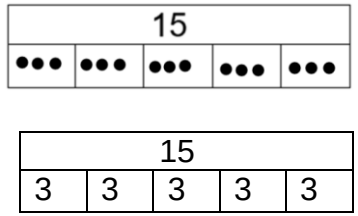
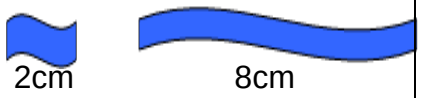
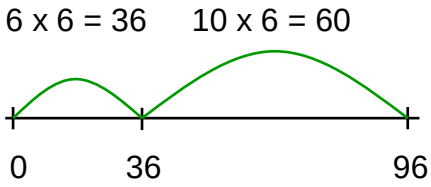
					 2 3 × 1 4 9 2 (23 × 4) 1 2 3 0 (23 × 10) 1 3 2 × 1 4 5 2 8 (132 × 4) 1 1 3 2 0 (132 × 10)

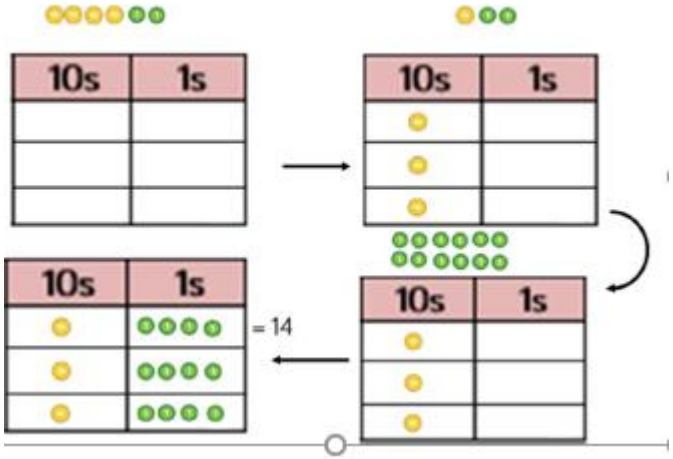
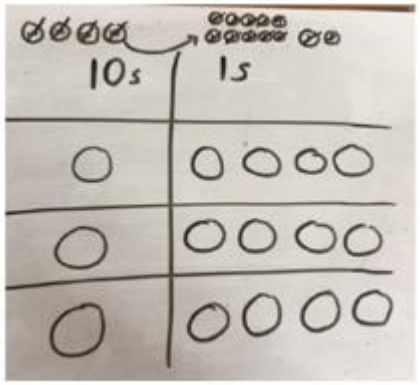
Stage 6:	Consolidate all previous counting, including forwards and backwards in fractions.	Perform mental calculations, including with mixed numbers and operations.	Recall & use multiplication facts for all times-tables up to 12 X 12.	Continue to use previous strategies to multiply 2 digit numbers by up to 4 digit numbers. Use place value counters to support multiplying decimals by 1 digit. Progressing to written method.	1.212 X 3  3.6 X 4 T O . t 3 . 6 X 4 2 . 4 (4 X 0.6) 1 2 . 0 (4 X 3) 1 4 . 4 (2.4 + 12)

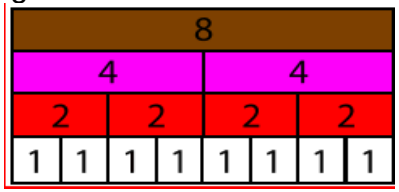
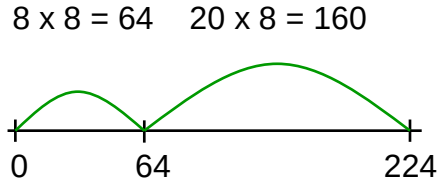
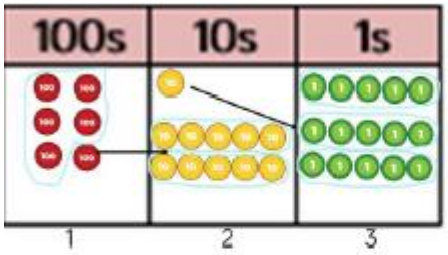
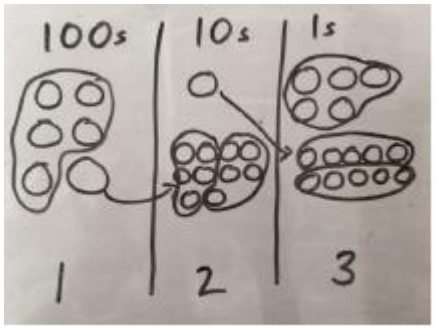
Division:

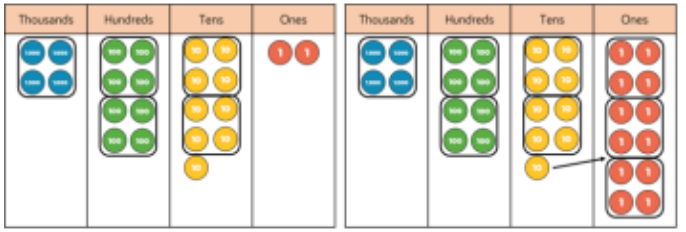
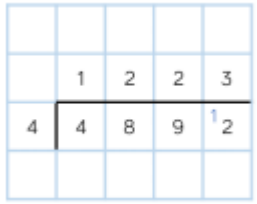
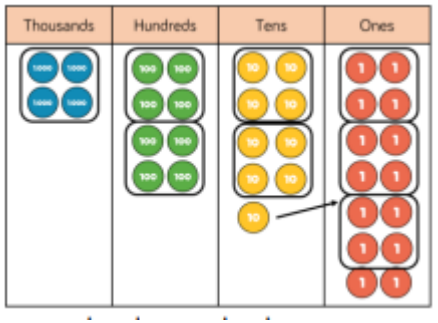
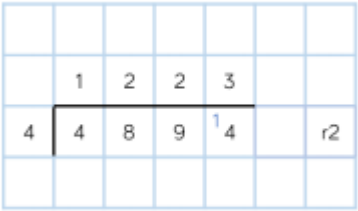
	Counting	Mental strategies	Rapid recall	Written calculation and appropriate models and images to support conceptual understanding	
Stage 1:	Count forwards and backwards in 2s, 5s and 10s	Doubling up to six and then ten whilst using related models and images.	Derive/recall doubles up to five and derive/recall halves up to ten.	**Developing early conceptual understanding of division as grouping and sharing:** Division as sharing Represent the sharing pictorially.	Use objects, pictorial representations and arrays to show the concept of division as grouping and sharing. 

				Division as grouping	 $6 \div 2 = 3$   Group 10 fish into equal groups of 2. How many groups are there? $10 \div 2 = 5$
Stage 2:	Count forwards and backwards in 2s, 3s, 5s and 10s from zero.	Begin to understand and use inverse number operations.  Stories are used alongside a triad to help children understand links between number operations, e.g. “15	Derive/recall doubles up to ten and derive/recall halves up to twenty. Recall and use multiplication facts for the 2X, 5X and 10X-tables.	Continue using methods in Stage 1 Use number lines to count up and see how many lots of a a number are in a given number.	$15 \div 5 = 3$  $15 \div 3 = 5$  $15 \div 5 = 3$

		children are asked to get into three groups and find out that there are five people in each group."		Use the bar model to support division by sharing	
Stage 3:	Counting forwards and backwards in 2s, 3s, 4s, 5s, 8s and 10s from zero.	Use doubling to make connections between the 2X, 4X and 8X-tables. Understand that multiplication can be undertaken by partitioning numbers, e.g. 12 X 4 = 10 X 4 + 2 X 4 Introduce the structure of scaling: e.g. Find a ribbon that is 4 times as long as the blue ribbon. 	Recall & use multiplication facts for the 2X, 3X, 4X, 5X, 8X and 10X tables.	**Dividing a 2-digit by 1-digit number, representing this efficiently on a number line:** Sharing using place value counters.	Children use an empty number line to chunk efficiently. 96 ÷ 6 = 16  Conceptual understanding can be provided through use of a bead string to highlight the chunks. 

				Children to represent the place value counters pictorially. Children to be able to make sense of the place value counters and write calculations that show the process.	42 ÷ 3 = 14   42 ÷ 3 42 = 30 + 12 30 ÷ 3 = 10 12 ÷ 3 = 4 10 + 4 = 14

Stage 4:	Counting forwards and backwards in 2s, 3s, 4s, 5s, 7s, 8s, 10s, 25s and 1000s from zero.	Derive factor pairs of numbers using models and images, e.g. Cuisenaire. 	Recall & use multiplication facts for all times-tables up to 12 X 12.	**Dividing a 3 or 2-digit by 1-digit number** Children to continue to represent this efficiently on a number line. Use place value counters to represent short division. Represent the place value counters pictorially.	224 ÷ 8 = 28  615 ÷ 5  1. Make 615 with place value counters. 2. How many groups of 5 hundreds can you make with 6 hundred counters? 3. Exchange 1 hundred for 10 tens. 4. How many groups of 5 tens can you make with 11 ten counters? 5. Exchange 1 ten for 10 ones. 6. How many groups of 5 ones can you make with 15 ones? 

Stage 5:	Counting forwards and backwards in 2s, 3s, 4s, 5s, 6s, 7s, 8s, 9s, 10s, 25s and 1000s from zero.	Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers.	Recall & use multiplication facts for all times-tables up to 12 X 12.	**Dividing a 4/3/2-digit by 1-digit number,** Division using place value counters progressing to short division Remainders should be interpreted in the following ways when long division is used: • as whole numbers • as fractions • through rounding in an appropriate way to the context	4892 ÷ 4   4894 ÷ 4  

Stage 6:	Consolidate all previous counting, including forwards and backwards in fractions.	Perform mental calculations, including with mixed numbers and different number operations.	Recall & use multiplication facts for all times-tables up to 12 X 12.	**Dividing a 4/3/2-digit by 2/1-digit number using long division.** Using place value counters Use of visual representations – like the ones opposite – remain important.